

**NONLINEAR CHARGE CONTROL DC AND TRANSMISSION  
LINE MODELS FOR GAAS MODFET's**

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**ABSTRACT**

A new simple analytical nonlinear charge control model is developed for two-dimensional electron gas density of GaAs MODFET and included in the d.c. model. MODFET is modeled as a lossy transmission line for microwave frequency analysis. The model predictions show a good agreement with experimental results of 0.3  $\mu\text{m}$  GaAs MODFET for both d.c. characteristics and 1-26 GHz frequency range.

**INTRODUCTION**

As the newly developed modulation doped field effect transistor(MODFET) has become an increasingly important device in low-noise microwave applications, the need for an accurate d.c. and a.c. models for a MODFET's is obvious. Although previous work has treated the d.c. characteristics of MODFET's, the high- frequency behavior was mainly analyzed experimentally[1]. This paper presents a physical analysis of MODFET's capable of providing information about their high frequency performance. A transmission line model is developed for microwave frequency analysis of MODEFT's. The a.c. analysis is preceded with d.c. analysis to determine the incremental resistance and transconductance of the transmission line. An improved d.c. model is developed by including a very simple new empirical formula for nonlinear charge control model.

**2DEG NONLINEAR CHARGE CONTROL MODEL**

The first step in modeling the behavior of a MODFET is to obtain a solution to the one-dimensional problem of charge control by a gate. In order to incorporate the effect of nonlinear charge control of 2DEG due to neutralization of donors in the AlGaAs layer, we propose the following piecewise expression for 2DEG concentration versus gate voltage:

$$n_{s1}(V_{gc}) = \frac{\epsilon_{AlGaAs}}{q(\Delta d + d_d + d_i)}(V_{gc} - V_{to}) \quad V_{gc} \leq V_p \quad (1)$$

$$n_{s2}(V_{gc}) = \frac{A(V_{gc} - V_{to})}{1 + B(V_{gc} - V_{to})} + C \quad V_{gc} > V_p \quad (2)$$

where  $\epsilon_{AlGaAs}$  is the dielectric constant of AlGaAs;  $d_i + d_d$ , the total thickness of AlGaAs layer as shown in Fig.1;  $V_{to}$  the threshold voltage;  $\Delta d = \epsilon_{AlGaAs}a/q \simeq 80(\text{\AA})$ ;  $a = 0.125 \times 10^{-12}(\text{eV}/\text{cm}^2)$ . The first equation describes the linear portion of concentration vs. gate voltage relationship as given in [2]. For the analysis of the saturation region caused by neutralizing of donors in AlGaAs layer as shown in Fig.2, we introduce an empirical equation(2), where A, B and C are the constants which can be determined analytically. There are two fitting parameters: 1).  $p$  ( $< 1$ ) indicates the point at which 2DEG is off the linear portion of the curve; 2).  $\Delta V_p$  is the voltage at which 2DEG equals to  $n_{so}$ , where  $n_{so}$  is the equilibrium concentration of the 2DEG.  $V_p$  can be determined by eq(1) as:

$$V_p = V_{to} + \frac{q(\Delta d + d_i)p n_{so}}{\epsilon_{AlGaAs}} \quad (3)$$

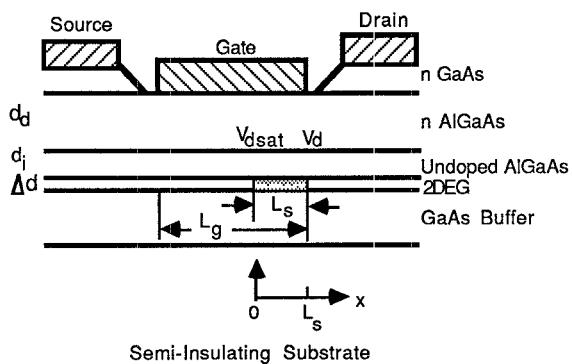


Fig.1 The GaAs MODFET structure

The three constants are determined by the following three boundary conditions: (1).  $n_{s1}(V_p) = n_{s2}(V_p)$ ; (2).  $\partial n_{s1}(V_p)/\partial V_g$

$\partial n_{s2}(V_p)/\partial V_g$ ; (3).  $n_{s2}(V_p + \Delta V_p) = n_{so}$ . After manipulation, the constants can be expressed as following:

$$A = \frac{\epsilon_{AlGaAs}}{q(d_i + d_d + \Delta d)} \left[ \frac{\eta \Delta V_p}{V_1(\eta - V_2) + V_2^2} \right]^2 \quad (4)$$

$$B = \frac{\Delta V_p - \eta}{V_1(\eta - V_2) + V_2^2} \quad (5)$$

$$C = n_{so} \left[ \frac{V_1(p\eta - V_2) + V_2^2}{V_1(\eta - V_2) + V_2^2} \right]^2 \quad (6)$$

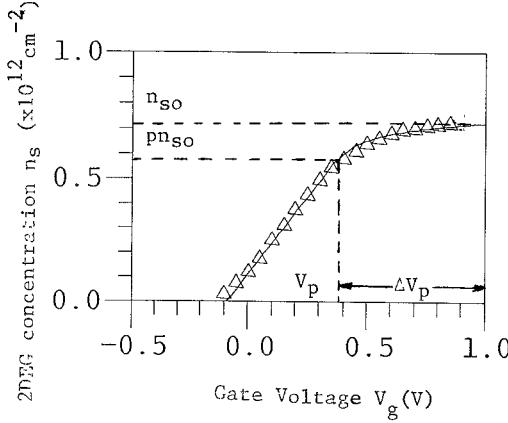


Fig.2 2DEG concentration versus gate-to-channel voltage for AlGaAs/GaAs MODFET. Solid line: two-piece analytical model based on eqn(1) and (2).  $\Delta$ : Fully numerical self-consistent two-band model[3]

where  $V_1 = V_p + \Delta V_p - V_{to}$ ;  $V_2 = V_p - V_{to}$ ;  $\eta = n_{so}(d_i + d_d + \Delta d)(1 - p)/\epsilon_{AlGaAs}$ . Fig.2 shows a very good agreement of our new model with fully numerical simulation result[3].

#### D.C. I-V CHARACTERISTICS

The d.c. analysis is conducted by using the above nonlinear charge control model together with the current continuity equation and the following empirical formula for electron velocity vs. electric field[4],

$$v = v_s [1 - \exp(-E(x)/E_s)] \quad (7)$$

From eqn(7) and the current continuity equation, one can derive the following equation:

$$dV = E_s \log \left[ 1 - \frac{I_{ds}}{qv_s W n_s(x)} \right] dx \quad (8)$$

In the above equation,  $n_s$  must be substituted by either eqn(1) or eqn(2) depending on the following different source-drain bias conditions:

Condition 1:  $V_s = V_g - V_{to} - I_{ds}R_s > V_p - V_{to}$  and  
 $V_g - V_{to} - V_d + I_{ds}R_d > V_p - V_{to}$

$$\int_{V_s}^{V_d} \frac{dV}{\log[1 - I_{ds}/qv_s n_{s1}(V)W]} = - \int_0^{L_s} E_s dx \quad (9a)$$

Condition 2:  $V_s = V_g - V_{to} - I_{ds}R_s > V_p - V_{to}$  and  
 $V_g - V_{to} - V_d + I_{ds}R_d \leq V_p - V_{to}$

$$\int_{V_s}^{V_p} \frac{dV}{\log[1 - I_{ds}/qv_s n_{s1}(V)W]} + \int_{V_p}^{V_d} \frac{dV}{\log[1 - I_{ds}/qv_s n_{s2}(V)W]} = - \int_0^{L_s} E_s dx \quad (9b)$$

Condition 3:  $V_s = V_g - V_{to} - I_{ds}R_s \leq V_p - V_{to}$  and  
 $V_g - V_{to} - V_d + I_{ds}R_d \leq V_p - V_{to}$

$$\int_{V_s}^{V_d} \frac{dV}{\log[1 - I_{ds}/qv_s n_{s2}(V)W]} = - \int_0^{L_s} E_s dx \quad (9c)$$

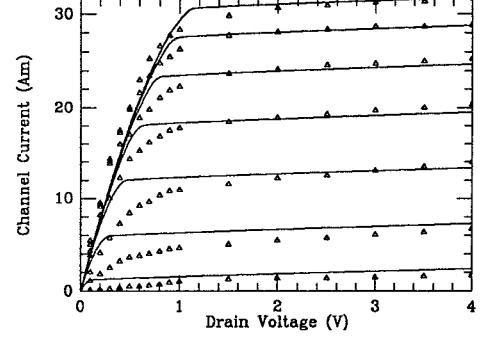


Fig.3 Comparison between the experimental and calculated  $I - V$  characteristics.  $\Delta$ : measurement data[5]; solid lines: calculations using model developed.

Equation (9) describes the output I-V characteristics of MODFET's. Given  $V_{ds}$  and  $V_g$ , eqn(9) can then be iteratively solved for  $I_{ds}$ . The theoretical curve by eqn(9) is compared to experimental data[5] as shown in Fig.3, which shows a reasonable agreement. The device parameters are listed in Table 1.

Table 1

$L_s = 1.0 \mu m$	$p = 0.70$
$W = 100 \mu m$	$\Delta V_p = 1.30V$
$d_d = 270 \text{ \AA}$	$E_s = 3.0 KV/cm$
$d_i = 30 \text{ \AA}$	$v_s = 2.25 \times 10^7 cm/sec$
$n_{so} = 0.92 \times 10^{12} cm^{-2}$	$R_s = 8.4 \Omega$
$V_{to} = -0.42V$	$R_d = 8.4 \Omega$

#### TRANSCONDUCTANCE

The expression for the most important parameter, transconductance  $g_m$ , can be derived by the method proposed by Chang and Fetterman [4] as follows:

Condition 1:  $V_s = V_g - V_{to} - I_{ds}R_s > V_p - V_{to}$  and  
 $V_g - V_{to} - V_d + I_{ds}R_d > V_p - V_{to}$

$$g_m = \frac{I_{ds}[1 - f(z_s, z_d)]}{R(z) + P(t_s)f(z_s, z_d) - P(t_d) + T(z)} \quad (10a)$$

Condition 2:  $V_s = V_g - V_{to} - I_{ds}R_s > V_p - V_{to}$  and  
 $V_g - V_{to} - V_d + I_{ds}R_d \leq V_p - V_{to}$

$$g_m = \frac{I_{ds}[1 - f(z_s, z_d)]}{R(z) + P(t_s)f(z_s, z_d) + t_pS(t_p) - t_d + T(z)} \quad (10b)$$

Condition 3:  $V_s = V_g - V_{to} - I_{ds}R_s \leq V_p - V_{to}$  and  
 $V_g - V_{to} - V_d + I_{ds}R_d \leq V_p - V_{to}$

$$g_m = \frac{I_{ds}[1 - f(z_s, z_d)]}{R(z) + t_s f(z_s, z_d) - t_d + \xi \log(1 - z_d)} \quad (10c)$$

where  $\xi = E_s(L_g - L_s)$ ;  $\delta = 1/qv_sW$ ;

$$D = \epsilon_{AlGaAs}/q(d_i + d_d + \Delta d);$$

$$t_s = V_g - V_{to}; t_p = V_p - V_{to}; t_d = V_g - V_{to} - V_{ds};$$

$$z_p = \delta I_{ds}/D t_p; z_d = I_{ds}/q v_s W n_s(t_d).$$

$$f(z_s, z_d) = \log(1 - z_d)/\log(1 - z_s);$$

$$f(z_p, z_d) = \log(1 - z_d)/\log(1 - z_p);$$

$$P(t) = (1 + Bt)[(A + BC)t + C]/A;$$

$$Q(z) = 2AB\delta/[z(A + BC) - \delta I_{ds}B]^2 \log(1 - z);$$

$$R(z) = R_s[f(z_s, z_d) + R_d/R_s]I_{ds}$$

$$S(t_p) = f(z_p, z_d)[1 - D(1 + Bt_p)^2/A].$$

$$T(z) = \log(1 - z_d)[\xi - \delta I_{ds}^2 \int_{z_s}^{z_d} Q(z) dz]$$

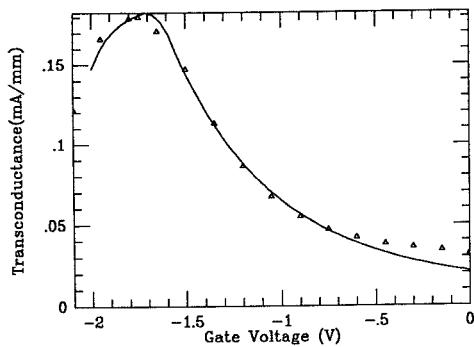


Fig.4 The calculated and experimental transconductance of Allied-signal 0.3  $\mu\text{m}$  MODFET as a function of the gate voltage.  $\Delta$  measurement data at  $V_d = 2.5\text{V}$ ; solid line: calculated curve.

Fig.4 shows the calculated result of the transconductance compared with the experimental measurement of an Allied-Signal 0.3  $\mu\text{m}$  GaAs MODFET, which shows a very good agreement. The device parameters are listed in Table 2.

Table 2

$L_g = 0.3 \mu\text{m}$	$p = 0.6$
$W = 100 \mu\text{m}$	$\Delta V_p = 0.80\text{V}$
$d_d = 300 \text{ A}$	$E_s = 2.0 \text{ KV/cm}$
$d_i = 30 \text{ A}$	$v_s = 1.13 \times 10^7 \text{ cm/sec}$
$n_{so} = 0.99 \times 10^{12} \text{ cm}^{-2}$	$V_{to} = -2.15\text{V}$

## TRANSMISSION LINE MODEL

The transmission line model for microwave frequency a.c. analysis is shown in Fig.5. For each increment  $j$  in Fig.5, the incremental resistance  $r_j$ , incremental capacitance  $c_j$  and incremental transconductance  $g_m$ , are as follows:  $c_j = C_o W dx$ ;  $r_j = [V_{j+1} - V_j]/I_{ds}$ ;  $g_m = \beta V_{j+1}/[dx/L_g + \alpha V_{j+1}]$ ,  $V_j$  is the d.c. voltage along the channel. At each node of Fig.5, Kirchoff's current law is used for small- signal a.c. analysis. For each node, the following equation is derived:

$$-\left(\frac{1}{r_j} + g_m\right)\tilde{v}_{j-1} + \left(\frac{1}{r_j} + \frac{1}{r_{j+1}} + jwc_{j+1} + g_{m+1}\right)\tilde{v}_j - \frac{1}{r_{j+1}}\tilde{v}_{j+1} = \tilde{v}_{gs} \left(jwc_{j+1} + g_{m+1} - g_m\right) \quad (11)$$

where  $\tilde{v}_j$  is the a.c. voltage, as shown in above equation, the a.c. voltage at each node depends on the a.c. voltage of the previous and the next node only. By applying Kirchoff's current law to every node along the channel, a set of tridiagonal linear equations can be obtained. The small-signal a.c. current and voltage distribution along the channel can be calculated by solving the tridiagonal linear equations using Gaussian elimination method. The terminal small- signal parameters are then determined by mixed parameters:

$$\tilde{i}_g = m_{11}\tilde{v}_{gs} + m_{12}\tilde{i}_d \quad (12)$$

$$\tilde{v}_{ds} = m_{21}\tilde{v}_{gs} + m_{22}\tilde{i}_d \quad (13)$$

where  $\tilde{i}$  and  $\tilde{v}$  are a.c. current and voltage respectively. By shorting the input circuit or opening the output circuit, the a.c. current and voltage along the channel are calculated by solving tridiagonal linear equations. M-parameters are then converted into S-parameter. The calculated S-parameter based on the transmission line model is compared with the measurement result of Allied-Signal 0.3  $\mu\text{m}$  MODFET as shown in Fig.6, which shows a close agreement. From Fig.6, one can see that the measured  $S_{21}$  is larger than the calculated results in high frequencies. This increase of the transconductance can be attributed to an effective decrease of the source access resistance at microwave frequency.

## CONCLUSION

An improved d.c. model for GaAs MODFET is developed which includes a very simple nonlinear charge control model for 2DEG. GaAs MODFET characteristics can be accurately modeled as a lossy transmission line. All the calculated results based on the new model agree with the experimental data very well.

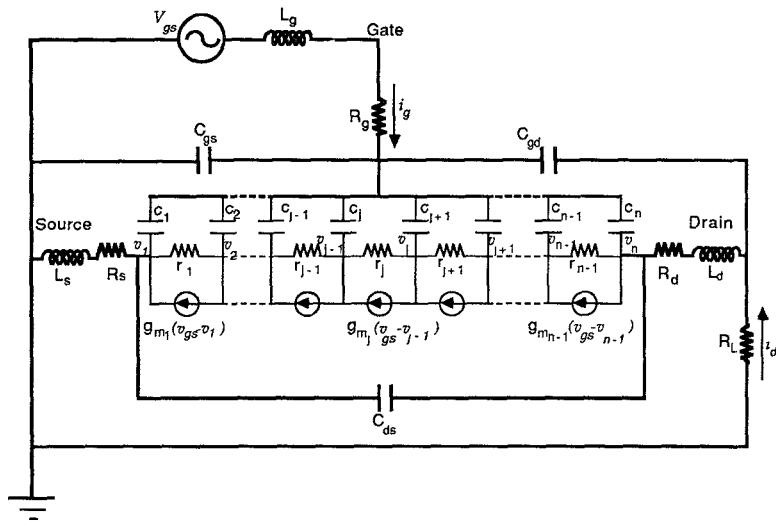


Fig.5 Transmission line model for MODFET

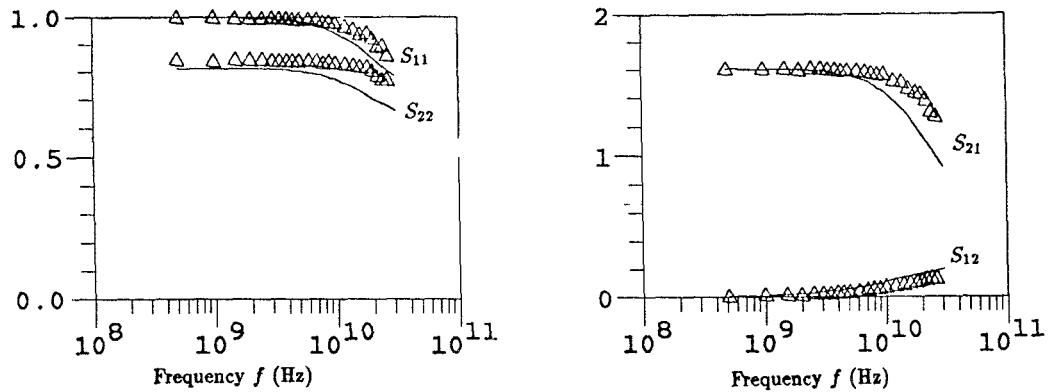


Fig.6. The calculated and experimental S-parameters (Magnitude).  $\Delta$ : experimental data; solid lines: calculations based on transmission line model.

#### ACKNOWLEDGMENT

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